July 1

Problem 1.

Let $\mathbf{v}_1, ..., \mathbf{v}_r$ be an orthonormal basis for a subspace W of $\mathbb{C}^n$. Write down the orthogonal projection onto W (call it Π), using ket-bra notation.

What about orthogonal projection onto $W^\perp$?

Problem 2.

Describe the possible outputs of the following programs, with the probabilities that they occur. (If the program outputs multiple times during its execution, show each full output sequence and the probability that output sequence occurs.)

(a) System is $\mathbb{C}^2$, with initial state $\mathbb{C}\,|\mathbf{e}_0\rangle$. The unitary U is represented by $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$.

> `Apply` U `to the current state.`
> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_0\rangle$`?`

(b) System is $\mathbb{C}^2$, with initial state $\mathbb{C}\,|\mathbf{e}_0\rangle$.

> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_0 + \mathbf{e}_1\rangle$`?`
> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_0\rangle$`?`

(c) System is $\mathbb{C}^3$, with initial state $\mathbb{C}\,|\mathbf{e}_0 + 2\mathbf{e}_1 + \mathbf{e}_2\rangle$.

> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_0\rangle$`?`
> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_1\rangle$`?`
> `Is the current state contained in` $\mathbb{C}\,|\mathbf{e}_2\rangle$`?`

Problem 3.

Using H and NOT gates, write a 1-qubit quantum circuit which maps inputs to outputs as follows

$$\begin{aligned}|0\rangle &\mapsto |0\rangle\\ |1\rangle &\mapsto -|1\rangle.\end{aligned}$$

The gate which has this behavior is called the Z gate.